Optimal Admission Control in Two-class Preemptive Loss Systems

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Abstract

We study optimal admission control in a two-class preemptive loss system. A class-1 customer arrival aborts service of a class-2 customer if the system is full upon arrival. Each successfully serviced class-2 customer leads to a reward, whereas each aborted class-2 customer incurs a cost. Using dynamic programming, we characterize optimal admission control for class-2 customers that maximizes the long-run average profit. The optimal admission control policy depends only on the total occupancy and is of threshold type.

Keywords: Revenue Maximization, Dynamic Programming, Markov Decision Process.

1. Introduction

In this paper, we study a preemptive loss system whose resources are regulated with admission control. We consider two classes of customers. Arrivals of each class are independent Poisson processes with rate $\lambda_i > 0$ for class-i customers where $i = 1, 2$. The system consists of C identical and parallel servers and service rates of all customers are independent and exponentially distributed with mean μ^{-1} unless terminated prematurely. Upon the departure of a successfully serviced class-2 customer, a fixed reward is earned.

We consider preemption-based prioritization between the two customer classes. In the context of a loss network, customers compete for a finite number of resources and preemption refers to aborting ongoing service of a low-priority customer for the sake of admitting a high-priority customer. Such implementation of preemption is widely used in engineering applications [1].

In our system, class-1 customers have preemptive priority over class-2 customers. Thus, a class-2 customer is removed from the system if the system is full (i.e. all C servers are busy) upon a class-1 customer arrival. When a class-2 customer is preempted, it is withdrawn from the system permanently, and a preemption cost is incurred. Such a model is studied in [2] where class-1 (2) customers are the primary users (secondary users) in a cognitive radio network. The aim is to find the optimal admission control policy of class-2 customers that maximizes the long-run average profit.

The related work can be classified in two categories: dynamic control of queueing systems with an emphasis on admission control and preemption.

Admission control of queueing systems and trunk reservation models have been widely studied [3]. Earlier work includes various seminal papers such as Miller [4] and Ramjee et al. [5], which consider a multi-class and multi-server queueing system that uses admission control to maximize the expected average return. Kelly [6] studies a trunk reservation model for networked resources. Recent studies include the work of Gans and Savin [7]. They characterize the optimal pricing policies in a system that consists of two types of customers. None of the mentioned work considers preemption.

One of the earliest works on preemption is the work of Helly [8], which proposes two approaches on the control of two-class traffic with different priorities and limited number of servers. Garay and Gopal [9] use preemption as a control mechanism in networks and outline the optimal preemption decision for greater revenue. Xu and Shanthikumar [10] present some results on optimal admission control by using duality of two systems one of which employs preemption. Brouns and van der Wal [11] study the optimal termination and admission controls in a two-class single server system. Brouns [12] extends these results to the multi-server case, where there are only queueing costs. In contrary to our system, their system does not have preemption cost per customer. Thereafter, Zhao et al. [1] utilize preemption to provide differentiated services to various classes in a parallel loss network. They compute the preemption probability and the preemption rate for each class analytically but they do not consider any type of admission control. Ulukus et al. [13] have the closest model to our system. They prove that preemption is optimal only when the system is full and they characterize the optimal termination and admission policies as state dependent threshold policies. Due to their statistical assumptions, they cannot derive a threshold type policy as descriptive as ours.

In this work, we formulate a two-dimensional (2D) Markov decision process (MDP) optimization problem. Our main contribution is to show that the optimal admission control policy is of threshold type. Similar to the well-studied admission control problem where customers are blocked when the system is full [4, 5], the optimal threshold for our system depends only on the total number of customers in the system, i.e. total occupancy.

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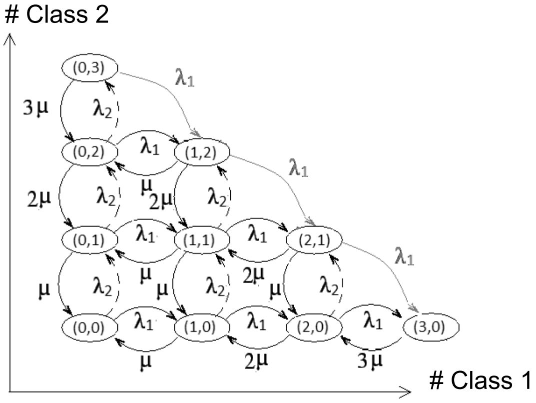

Figure 1: State diagram of 2D Markov chain for $C = 3$. Depending on the admission control, the dashed transitions with rate λ_2 upon class-2 arrivals may exist or not.

This conclusion is non-trivial despite the extensive literature on optimality of threshold admission policies for *non-preemptive* loss systems (see, for example, [4] for a seminal paper on this issue). Conclusions concerning non-preemptive systems do not immediately extend to preemptive systems because in a non-preemptive system, customers cannot be evicted once they are admitted, and in turn, due to identical service rate statistics, customers are indistinguishable once they are in the system. In contrast, a preemptive system entails an asymmetrical situation since admission of a class-1 customer when the system is full depends on the presence of class-2 customers in the system at the time of its arrival. Hence, it is not clear from the outset whether analysis of optimal profit requires consideration of a multi-dimensional model.

Interestingly, although optimal admission control policies have the same form for non-preemptive and preemptive systems with memoryless service times, differences emerge when this latter condition is relaxed: On the one hand, it is known that for a two-class non-preemptive system with general service time distribution, the optimal occupancy-based policy, i.e. the policy that depends only on the number of customers of *each* class, is of threshold type [14]. On the other hand, we show here by an example that for preemptive systems threshold type policies are not necessarily optimal even within occupancy-based policies under general service time distributions.

The rest of the paper is organized as follows: We describe our system model in Section 2. The model analysis and characterization of admission control are given in Section 3. Finally, we discuss general service time distributions in Section 4.

2. Model Description

Knowing that the inter-arrival and service times are exponentially distributed and the service of a class-2 customer may be interrupted at any time instant, the system behaves as a 2D continuous-time MDP. An example state transition diagram for 2D MDP with $C = 3$ is given in Fig. 1. The system description is as follows:

States: The state of the system is in the form of a tuple (x, y) where $x \geq 0$ is the number of class-1 customers and $y \geq 0$ is the number of class-2 customers in the system.

Decisions: The only decision control is the admission control of the class-2 customers. Upon a class-2 arrival, the system either accepts or rejects the customer. Class-1 customers are always admitted if there are less than C class-1 customers in the system.

Rewards and costs: $r > 0$ is the reward collected from a class-2 customer per unit time. Then, $R = r/\mu$ is the average reward per class-2 customer, which is collected after a class-2 customer leaves the system with successful completion of service. Blocking customers upon arrival is free of charge. The system preempts a class-2 customer whenever a class-1 customer arrives and finds the system full and there exists a class-2 customer in the system. Employing preemption is optimal only when all C servers are busy [13, 15]. For every preempted class-2 customer, a cost $K > 0$ is incurred.

Discounting: When we use discounting at a rate $\alpha \geq 0$, the rewards and costs at time t are scaled by a factor of $exp(-\alpha t)$. In other words, the reward gained in the present is more valuable than the reward gained in the future [16]. A process with an exponential discount rate is equivalent to a process *killed* at an exponential rate α [17]. There are no arrivals, departures, preemptions, rewards or costs after the process vanishes [12].

Uniformization: The process we have is a continuous-time Markov chain. We develop the discrete-time equivalent of this system using a uniformization technique [18]. Without loss of generality, we set the maximum possible rate out of any state to 1 (i.e. $\lambda_1 + \lambda_2 + C\mu + \alpha = 1$). Hence, a class-1 (2) arrival occurs with probability λ_1 (λ_2), a class-1 (2) departure occurs with probability $x\mu$ ($y\mu$), the process terminates with probability α and the system stays at the same state with probability $(1 - \lambda_1 - \lambda_2 - x\mu - y\mu - \alpha) = (C - x - y)\mu$.

3. Model Analysis and Characterization of the Admission Control Policy

In this section, we formulate the average profit rate of the system and determine a method to maximize it through the optimal admission control of class-2 customers. We define S as the state space for all states given that there are C channels, that is $S = \{(x, y) | x+y \leq C, \forall x, y \geq 0\}$. Let $S_1 \subset S$ be the sub-space of preemptive states. Formally, $S_1 = \{(x, y) \mid x + y = C,\ y \geq 1\}$. We define $\pi_p(x, y)$ as the steady state probability that the system is in state (x, y) under policy p. In this scheme, J_p the average profit rate under policy p is as follows:

$$J_p = R \sum_{(x,y)\in S} y\mu\pi_p(x, y) - K\lambda_1 \sum_{(x,y)\in S_1} \pi_p(x, y). \qquad (1)$$

The first term in Eq. (1) corresponds to the total average revenue rate collected from class-2 customers and the second term is the total average cost due to the preempted class-2 customers. Their difference yields the total average profit rate.

In order to find the optimal admission control policy p^* of class-2 customers that maximizes the average profit rate J_p in

Eq. (1) and yields the optimal profit J^*, the problem can be formulated as a *stochastic dynamic programming (DP) problem* [16].

3.1. Dynamic Programming Formulation

In this section, we formulate the problem explicitly and provide a solution using the DP algorithm. Instead of the average profit, we derive the maximum finite horizon discounted profit to be able to utilize useful analysis techniques such as induction. Then, we extend our findings to the infinite horizon average profit. First, we reverse the time index n and define it as the observation points left until the end of the horizon. We define the profit function at this point.

Definition 1. *$V_n(x, y)$ is the maximal expected discounted profit for the system in the current state (x, y) at time period n.*

The corresponding DP equations are as follows:

For $n = 0$: $V_0(x, y) = 0 \quad$ for $\quad x, y \geq 0$

For $n \geq 1$:

• $x + y < C$

$$\begin{aligned} V_n(x, y) = {} & \lambda_1 V_{n-1}(x + 1, y) \\ & + \lambda_2 \max\{V_{n-1}(x, y), V_{n-1}(x, y + 1)\} \\ & + x\mu V_{n-1}(x - 1, y) + y\mu[V_{n-1}(x, y - 1) + R] \\ & + (C - x - y)\mu V_{n-1}(x, y) \end{aligned} \tag{2}$$

• $x = C$ and $y = 0$

$$V_n(x, y) = \lambda_1 V_{n-1}(x, y) + \lambda_2 V_{n-1}(x, y) + C\mu V_{n-1}(x - 1, y)$$

• $x \neq C$ and $x + y = C$

$$\begin{aligned} V_n(x, y) = {} & \lambda_1[V_{n-1}(x + 1, y - 1) - K] + \lambda_2 V_{n-1}(x, y) \\ & + x\mu V_{n-1}(x - 1, y) + y\mu[V_{n-1}(x, y - 1) + R]. \end{aligned}$$

We set $V_n(-1, y) = V_n(0, y)$ and $V_n(x, -1) = V_n(x, 0)$ when required. Note that there are three different equations in the DP formulation. The first DP equation is for the case when there are idle channels in the system for the use of all customers. The second equation is for the case when all channels are used by class-1 customers and there are no class-2 customers in the system. Lastly, the third equation is for the preemption case.

3.2. Characteristics of the Optimal Admission Control Policy

In this subsection, we characterize the optimal admission control policy of class-2 customers.

Theorem 1. *The optimal admission control policy p^* of class-2 customers is of threshold type and it depends only on the total number of customers in the system. Thus, there exists an optimal threshold T^* such that if a class-2 arrival finds the system in state (x, y) and if $x + y < T^*$, it is accepted. Otherwise, it is rejected.*

Suppose we have two states on the same diagonal i.e., the total occupancy is equal but the number of class-2 customers and the number of class-1 customers are different individually. Theorem 1 declares that the states that have the same total number of customers must have the same admission decision upon an arriving class-2 customer. To prove this claim, we first prove certain monotonicity properties of the system which are given in the following lemmas.

The following lemma sets a lower bound on the value of an additional class-2 customer. It can be deduced from Lemma 1 in [13] where the authors study a more general model with non-identical service rates. For the sake of completeness, we provide a full proof in the present setting.

Lemma 1. *For all (x, y) with $x + y + 1 \leq C$ and $\forall n \geq 0$:*

$$V_n(x, y + 1) - V_n(x, y) \geq -K. \tag{3}$$

Proof. We prove this statement by a sample path argument. Let System A and System B be two coupled systems except for an additional class-2 customer in System A. All customers are identical other than the extra customer. System A starts at state $(x, y + 1)$ where System B starts at state (x, y). Furthermore, the inter-arrival times and service times are the same for the systems. Assume that System B follows the optimal admission control policy and System A imitates all decisions of System B, i.e. if System B accepts (rejects) a class-2 customer, System A also accepts (rejects) it.

We consider two critical cases which may alter the difference between the profits of the systems. In the first case, the additional class-2 customer successfully leaves the system before the total number of customers in System A reaches C, which causes System A to earn reward R. In the second case, the total number of customers in System A reaches C before the additional customer departs. Then, System B has $C - 1$ customers in total. If a class-2 customer arrives, System A must reject this customer whereas System B may accept it. Then, the systems are in the same state after this point and have the same profit. If a class-1 arrival occurs, however, System A preempts a class-2 customer by paying a cost K. System B may accept the arriving class-1 customer and the systems couple. For all cases, Eq. (3) holds which proves the lemma. □

The next lemma declares that the value of an additional class-2 customer is non-increasing in the number of class-1 customers x for fixed number of class-2 customers y in the system.

Lemma 2. *For all (x, y) with $x + y + 2 \leq C$ and $\forall n \geq 0$:*

$$V_n(x, y + 1) - V_n(x, y) \geq V_n(x + 1, y + 1) - V_n(x + 1, y). \tag{4}$$

Proof. This lemma can be proven by induction on n, the number of periods left in the horizon.

Step 1: Inequality (4) holds for $n = 0$ by definition.

Induction step: Assume that for $n \geq 0$, inequality (4) holds.

Step 2: Assuming (4) holds for n, we show that it holds for $n + 1$ as well. There are two cases to consider. First, all the states in inequality (4) are non-preemptive states, i.e. $0 \leq x + y < C - 2$. Second, state $(x + 1, y + 1)$, which has the greatest total number of customers, is a preemptive state. This time, $x + y = C - 2$. We make these distinctions because the corresponding DP equations depend on the type of the state. We substitute the DP equations and use term by term comparison as follows:

Case 1. $0 \le x + y < C - 2$

$$
\begin{aligned}
&V_{n+1}(x, y+1) - V_{n+1}(x, y) \\
&= \lambda_1[V_n(x+1, y+1) - V_n(x+1, y)] \text{①} \\
&\quad \left.\begin{aligned} &+ \lambda_2[\max\{V_n(x, y+1), V_n(x, y+2)\} \\ &- \max\{V_n(x, y), V_n(x, y+1)\}] \end{aligned}\right\} \text{②} \\
&\quad \left.\begin{aligned} &+ [x\mu[V_n(x-1, y+1) - V_n(x-1, y)] \\ &+ \mu[V_n(x, y+1) - V_n(x, y)]] \end{aligned}\right\} \text{③} \\
&\quad + y\mu[V_n(x, y) - V_n(x, y-1)] \text{④} \\
&\quad + (C - x - y - 2)\mu[V_n(x, y+1) - V_n(x, y)] + \mu R \text{⑤} \\
&\ge \lambda_1[V_n(x+2, y+1) - V_n(x+2, y)] \text{❶} \\
&\quad \left.\begin{aligned} &+ \lambda_2[\max\{V_n(x+1, y+1), V_n(x+1, y+2)\} \\ &- \max\{V_n(x+1, y), V_n(x+1, y+1)\}] \end{aligned}\right\} \text{❷} \\
&\quad + (x+1)\mu[V_n(x, y+1) - V_n(x, y)] \text{❸} \\
&\quad + y\mu[V_n(x+1, y) - V_n(x+1, y-1)] \text{❹} \\
&\quad + (C - x - y - 2)\mu[V_n(x+1, y+1) - V_n(x+1, y)] + \mu R \text{❺} \\
&= V_{n+1}(x+1, y+1) - V_{n+1}(x+1, y).
\end{aligned}
$$

At this point, we explain the relations between each term pair to complete the proof for Case 1. The relations ① ≥ ❶, ③ ≥ ❸, ④ ≥ ❹ and ⑤ ≥ ❺ are direct consequences of the induction hypothesis (4). Inequality ② ≥ ❷ is more complicated and requires further justification.

•② ≥ ❷

$$
\begin{aligned}
&\lambda_2 \max\{V_n(x, y+1), V_n(x, y+2)\} \\
&\quad - \lambda_2 \max\{V_n(x, y), V_n(x, y+1)\} && (5) \\
&= \lambda_2[\max\{V_n(x, y+1), V_n(x, y+2)\} - V_n(x, y+1)] \\
&\quad - \lambda_2[\max\{V_n(x, y), V_n(x, y+1)\} - V_n(x, y+1)] && (6) \\
&= \lambda_2 \max\{V_n(x, y+2) - V_n(x, y+1), 0\} \\
&\quad + \lambda_2 \min\{V_n(x, y+1) - V_n(x, y), 0\} && (7) \\
&\overset{(4)}{\ge} \lambda_2 \max\{V_n(x+1, y+2) - V_n(x+1, y+1), 0\} \\
&\quad + \lambda_2 \min\{V_n(x+1, y+1) - V_n(x+1, y), 0\} \\
&= \lambda_2[\max\{V_n(x+1, y+1), V_n(x+1, y+2)\} \\
&\quad - V_n(x+1, y+1)] \\
&\quad - \lambda_2[\max\{V_n(x+1, y), V_n(x+1, y+1)\} - V_n(x+1, y+1)] \\
&= \lambda_2 \max\{V_n(x+1, y+1), V_n(x+1, y+2)\} \\
&\quad - \lambda_2 \max\{V_n(x+1, y), V_n(x+1, y+1)\}.
\end{aligned}
$$

We add term $V_n(x, y+1)$ to (5) and subtract the same term in order to obtain (6). Then we change the *max* to *min* to get (7). When the same procedure is applied backwards to the other side of the inequality, we verify that ② ≥ ❷ holds.

Case 2. $x + y = C - 2$

$$
\begin{aligned}
&V_{n+1}(x, y+1) - V_{n+1}(x, y) \\
&= \lambda_1[V_n(x+1, y+1) - V_n(x+1, y)] \text{①} \\
&\quad \left.\begin{aligned} &+ \lambda_2[\max\{V_n(x, y+1), V_n(x, y+2)\} \\ &- \max\{V_n(x, y), V_n(x, y+1)\}] \end{aligned}\right\} \text{②} \\
&\quad \left.\begin{aligned} &+ x\mu[V_n(x-1, y+1) - V_n(x-1, y)] \\ &+ \mu[V_n(x, y+1) - V_n(x, y)] \end{aligned}\right\} \text{③} \\
&\quad + y\mu[V_n(x, y) - V_n(x, y-1)] + \mu R \text{④} \\
&\ge \lambda_1[V_n(x+2, y) - V_n(x+2, y)] - K\lambda_1 \text{❶} \\
&\quad \left.\begin{aligned} &+ \lambda_2[V_n(x+1, y+1) \\ &- \max\{V_n(x+1, y), V_n(x+1, y+1)\}] \end{aligned}\right\} \text{❷} \\
&\quad + (x+1)\mu[V_n(x, y+1) - V_n(x, y)] \text{❸} \\
&\quad + y\mu[V_n(x+1, y) - V_n(x+1, y-1)] + \mu R \text{❹} \\
&= V_{n+1}(x+1, y+1) - V_{n+1}(x+1, y).
\end{aligned}
$$

① ≥ ❶ is true from Lemma 1. ③ ≥ ❸ and ④ ≥ ❹ can be directly proven using inequality (4). However, we must show that ② ≥ ❷ holds as it is not an intuitive argument.

•② ≥ ❷

$$
\begin{aligned}
&\lambda_2 \max\{V_n(x, y+1), V_n(x, y+2)\} \\
&\quad - \lambda_2 \max\{V_n(x, y), V_n(x, y+1)\} \\
&= \lambda_2 \max\{V_n(x, y+2) - V_n(x, y+1), 0\} \\
&\quad + \lambda_2 \min\{V_n(x, y+1) - V_n(x, y), 0\} \\
&\overset{(4)}{\ge} \lambda_2[0 + \min\{V_n(x+1, y+1) - V_n(x+1, y), 0\}] \\
&= \lambda_2[V_n(x+1, y+1) - V_n(x+1, y+1)] \\
&\quad - \lambda_2[\max\{V_n(x+1, y), V_n(x+1, y+1)\} - V_n(x+1, y+1)] \\
&= \lambda_2 V_n(x+1, y+1) - \lambda_2 \max\{V_n(x+1, y), V_n(x+1, y+1)\}.
\end{aligned}
$$

□

The following lemma states that values of an additional class-2 customer at two neighbor states on the same diagonal, say $(x+1, y)$ and $(x, y+1)$, are equal. Consequently, all the states on the same diagonal (i.e. with the same total number of customers) have the same reward upon accepting a class-2 customer under the optimal policy.

Lemma 3. *For all* (x, y) *with* $x + y + 2 \le C$ *and* $\forall n \ge 0$*:*

$$V_n(x+1, y+1) - V_n(x+1, y) = V_n(x, y+2) - V_n(x, y+1). \quad (8)$$

Proof. Similar to Lemma 2, Eq. (8) is proven by induction.
Step 1: Eq. (8) holds for $n = 0$ by definition.

Induction step: Assume that for $n \ge 0$, Eq. (8) holds.
Step 2: Assuming Eq. (8) holds for n, we show that it holds for $n + 1$. In the first case we consider, all the states in Eq. (8) are non-preemptive states, i.e. $0 \le x + y < C - 2$. In the second case, states $(x+1, y+1)$ and $(x, y+2)$ are preemptive states. Thus, $x + y = C - 2$. The states have different DP equations depending on their nature.

We find the profit functions at $n + 1$ in terms of the profit functions at n by substituting the DP equations. Similar to the proof of Lemma 2, we group and label the terms and show that each term on the left-hand side has a corresponding term on the right-hand side of equal value.

Case 1. $0 \le x + y < C - 2$

$$
\begin{aligned}
&V_{n+1}(x+1, y+1) - V_{n+1}(x+1, y) \\
&= \lambda_1[V_n(x+2, y+1) - V_n(x+2, y)]\text{①} \\
&\quad \left.\begin{aligned} &+ \lambda_2[\max\{V_n(x+1, y+1), V_n(x+1, y+2)\} \\ &- \max\{V_n(x+1, y), V_n(x+1, y+1)\}] \end{aligned}\right\}\text{②} \\
&\quad + (x+1)\mu[V_n(x, y+1) - V_n(x, y)]\text{③} \\
&\quad + y\mu[V_n(x+1, y) - V_n(x+1, y-1)]\text{④} \\
&\quad + (C - x - y - 2)\mu[V_n(x+1, y+1) - V_n(x+1, y)] + \mu R\text{⑤} \\
&= \lambda_1[V_n(x+1, y+2) - V_n(x+1, y+1)]\text{❶} \\
&\quad \left.\begin{aligned} &+ \lambda_2[\max\{V_n(x, y+2), V_n(x, y+3)\} \\ &- \max\{V_n(x, y+1), V_n(x, y+2)\}] \end{aligned}\right\}\text{❷} \\
&\quad \left.\begin{aligned} &+ x\mu[V_n(x-1, y+2) - V_n(x-1, y+1)] \\ &+ \mu[V_n(x, y+1) - V_n(x, y)] \end{aligned}\right\}\text{❸} \\
&\quad + y\mu[V_n(x, y+1) - V_n(x, y)]\text{❹} \\
&\quad + (C - x - y - 2)\mu[V_n(x, y+2) - V_n(x, y+1)] + \mu R\text{❺} \\
&= V_{n+1}(x, y+2) - V_{n+1}(x, y+1).
\end{aligned}
$$

The equalities ① = ❶, ③ = ❸, ④ = ❹ and ⑤ = ❺ are direct consequences of Eq. (8). Now, we prove ② = ❷.

•② = ❷

$$
\begin{aligned}
&\lambda_2 \max\{V_n(x+1, y+1), V_n(x+1, y+2)\} \\
&\quad - \lambda_2 \max\{V_n(x+1, y), V_n(x+1, y+1)\} \\
&\overset{(8)}{=} \lambda_2 \max\{V_n(x, y+3) - V_n(x, y+2), 0\} \\
&\quad + \lambda_2 \min\{V_n(x, y+2) - V_n(x, y+1), 0\} \\
&= \lambda_2 \max\{V_n(x, y+2), V_n(x, y+3)\} \\
&\quad - \lambda_2 \max\{V_n(x, y+1), V_n(x, y+2)\}.
\end{aligned}
$$

Case 2. $x + y = C - 2$

$$
\begin{aligned}
&V_{n+1}(x+1, y+1) - V_{n+1}(x+1, y) \\
&= \lambda_1[V_n(x+2, y) - K - V_n(x+2, y)]\text{①} \\
&\quad + \lambda_2[V_n(x+1, y+1) - \max\{V_n(x+1, y), V_n(x+1, y+1)\}]\text{②} \\
&\quad + (x+1)\mu[V_n(x, y+1) - V_n(x, y)]\text{③} \\
&\quad + y\mu[V_n(x+1, y) - V_n(x+1, y-1)] + \mu R\text{④} \\
&= \lambda_1[V_n(x+1, y+1) - K - V_n(x+1, y+1)]\text{❶} \\
&\quad + \lambda_2[V_n(x, y+2) - \max\{V_n(x, y+1), V_n(x, y+2)\}]\text{❷} \\
&\quad \left.\begin{aligned} &+ x\mu[V_n(x-1, y+2) - V_n(x-1, y+1)] \\ &+ \mu[V_n(x, y+1) - V_n(x, y)] \end{aligned}\right\}\text{❸} \\
&\quad + y\mu[V_n(x, y+1) - V_n(x, y)] + \mu R\text{❹} \\
&= V_{n+1}(x, y+2) - V_{n+1}(x, y+1).
\end{aligned}
$$

Note that ① = ❶ = $-K\lambda_1$. ③ = ❸ and ④ = ❹ are directly proven using Eq. (8). At this point, we show that ② = ❷.

•② = ❷

$$
\begin{aligned}
&\lambda_2 V_n(x+1, y+1) - \lambda_2 \max\{V_n(x+1, y), V_n(x+1, y+1)\} \\
&\overset{(8)}{=} \lambda_2[0 + \min\{V_n(x, y+2) - V_n(x, y+1), 0\}] \\
&= \lambda_2[V_n(x, y+2) - \max\{V_n(x, y+1), V_n(x, y+2)\}].
\end{aligned}
$$

□

The next corollary is a natural result of Lemma 2 and Lemma 3. Adding Eq. (4) to Eq. (8), we obtain Eq. (9). The following corollary states that the profit function is concave in the number of class-2 customers.

Corollary 1. *For all* (x, y) *with* $x + y + 2 \le C$ *and* $\forall n \ge 0$*:*

$$V_n(x, y+1) - V_n(x, y) \ge V_n(x, y+2) - V_n(x, y+1). \quad (9)$$

Theorem 1 can be inferred from Lemma 3 and Corollary 1. For the states with the same total number of customers to have the same admission control decision, the value of an additional accepted class-2 customer must be identical. Lemma 3 states that the rewards gained by accepting an additional class-2 customer to the states with the same total number of customers are equal. Therefore, Lemma 3 implies that the states with the same total number of customers have the same optimal admission control decision.

Furthermore, the optimal admission control policy is of threshold type by Corollary 1. In the DP equations, Eq. (2) designates the admission control decision on the class-2 customers. The optimal decision d^* in state (x, y) at time period n is as follows:

$$d^* = \begin{cases} \text{accept} & \text{if } V_{n-1}(x, y+1) - V_{n-1}(x, y) \ge 0 \\ \text{reject} & \text{otherwise.} \end{cases}$$

For small values of y, it is optimal to accept class-2 customers until the optimal threshold value T^* is reached. Then, for larger values of y such that $x + y \ge T^*$, we always reject class-2 customers as the profit function is concave in y.

Our results so far are proven to hold for all $n \ge 0$. We can extend these α-discounted finite horizon results to the infinite horizon by taking $n \to \infty$, i.e. $V(x, y) = \lim_{n\to\infty} V_n(x, y)$. We refer to [19] for the conditions that make this extension possible. In addition, the properties of the optimal admission control policy hold for the long-run average return case since the control space and state spaces are finite. The average return case corresponds to $\alpha \to 0$ [13]. Thus, Theorem 1 holds for both the infinite horizon discounted return and average return formulations. We present an example to illustrate Theorem 1.

Example 1. We set $C = 5$, $\mu = 1$, $\lambda_1 = 3$, $\lambda_2 = 1$, $K = 10$ and $R = 5$. We determine the optimal admission control policy that maximizes the long-run average profit, i.e., $\alpha = 0$. The simulation results are illustrated in Fig. 2. The states with the same total number of customers have the same optimal admission control decision. Furthermore, the optimal policy is of threshold type where the optimal threshold is $T^* = 3$.

As a final remark, notice that our results also apply to systems that incur a cost per each blocked class-2 customer, as discussed in [20].

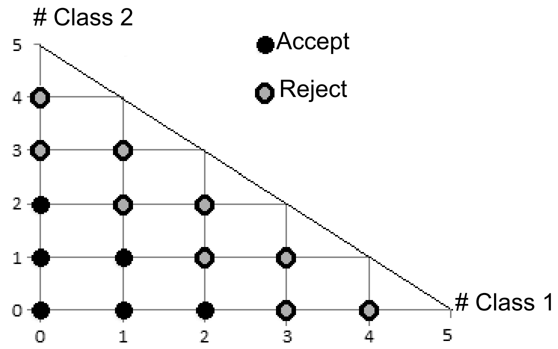

Figure 2: The optimal admission control decisions at each state of the 5 channel system in Example 1. The states with the same total occupancy have the same optimal decision and the optimal admission control policy is of threshold type.

4. Applicability of Threshold Property to General Service Time Distributions

In the previous sections, we have examined a system with exponentially distributed service times with mean μ^{-1}. In this section, we investigate whether the threshold structure holds for general service time distributions.

In earlier work [14], a two-class non-preemptive loss system which employs an occupancy-based policy is studied. All customers have the same general service time distribution. The system operator pays a cost when a class-1 customer is blocked due to class-2 customers. In [14], the optimal occupancy-based policy depends only on the total occupancy for general service time distributions. Through a counterexample, we show that for a preemptive system, the threshold property of the optimal occupancy-based admission control policy proven in Theorem 1 cannot be extended to general service time distributions.

Example 2. We assume that all customers have hypo-exponential service time distribution with two phases. In the first phase, the service time is exponentially distributed with mean $1/\mu_a$ and the customer proceeds to the second phase where the service time is exponentially distributed with mean $1/\mu_b$. We determine the optimal occupancy-based admission control policy of a system with $C = 2$, $\lambda_1 = 0.5$, $\lambda_2 = 0.2$, $\mu_a = 1/3$, $\mu_b = 2/3$, $K = 9$ and $R = 3$. We observe that the optimal admission decision for state (0,1) is *reject* whereas it is *accept* for state (1,0). The optimal admission control decision at the given states had to be identical for the optimal admission control policy to depend only on the total occupancy. Hence, unlike non-preemptive loss systems, the threshold structure does not hold for general distributions in preemptive systems.

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